GEOMETRY AND THE INTERNET

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Over the years I have found myself using the Internet more and more in my teaching in a variety of ways. In particular, I have used it extensively both in a standard geometry course and in a course I teach as an elective, The Mathematics of Patterns. In the former, I use it primarily to supplement the text. In the latter, the Internet is the source of all information for the course. I should add that I also use it extensively for my own research. In what follows, a site is listed along with a brief description of how it can be used and, where appropriate, a sample of what you might find there. The sites I have chosen fall into one of the following categories; general information, specific course content, or useful software and applets. I have also posted an electronic version of this article with links on my Website [www.zebragraph.com] so that you can easily check out the sites I discuss.

General Information Sites

The Math Forum: [http://mathforum.org] The Math Forum is by far the best general information site available. It has material for all levels of the curriculum and has lots of useful features that include:

- A powerful search engine.
- A Problem of The Week section with problems for most of the subjects taught in the pre-college curriculum.
- A variety of interesting ongoing discussions. The most recent one focused on the best computer-based drawing tools and contains some URLs for sites that provide helpful information on how best to use Word's Draw feature.
- Math Tools, an easy-to-use comprehensive collection of activities, demonstration applets, software (both free and for pay), and a variety of other on-line resources that can be used throughout the mathematics curriculum. There is also a useful Getting Started section here.

For example, if you go to the Math Forum, select Math Tools and then search for Law of Cosines you will find an applet that gives a neat visual proof of the Law of Cosines that uses the diagram in **Figure 1** and some clever use of transformations.

To see the actual proof, go to http://mathforum.org/mathtools and search for Law of Cosines.

MathWorld:
[http://mathworld.wolfram.com] If you are looking for information about any mathematical topic you should visit Wolfram's MathWorld site that Eric Weiisman has put together and do a search there. The site is easy to use and loaded with interesting material. For example, suppose you want to know more about the Law of Cosines. You can go to http://mathworld.wolfram.com/LawofCosines.html and you will find two proofs of the Law of Cosines, the Law of Cosines for spherical triangles, a generalized Law of Cosines for similar triangles, the Law of Cosines for tetrahedra, and links to material about the Law of Sines and the Law of Tangents. I found these two identities below at this site. Have fun proving them.

$a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) = 0$

$a = b\cos C + c\cos B$

Mathematical Association of America
[http://www.maa.org] This Website has lots of material of interest to secondary mathematics teachers. Ivars Petersen and Keith Devlin write columns that always contain interesting mathematics. Ivars

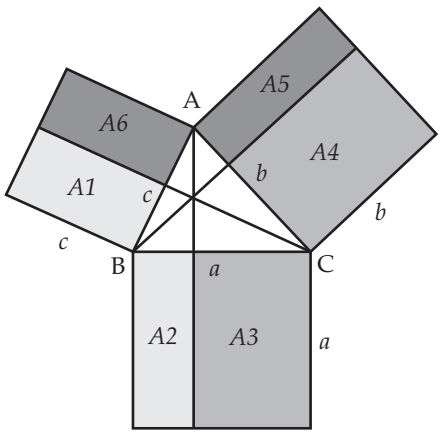

FIGURE 1.

Petersen's Mathematical Tourist column recently had a fascinating piece on bi-fold closet doors and how they take up less space than regular closet doors. Keith Devlin's recent Devlin's Angle column featured interesting material inspired by NUMB3RS, the popular TV show. He develops a nice example showing how mathematics is used in solving crimes.

Specific Course Content

David Joyce's Website
[http://aleph0.clarku.edu/~djoyce/home.html]: If you teach trigonometry in some form, you should check out Dave's Short Trig Course. It covers the basics of triangle trigonometry including lots of interesting historical background. Many of the concepts he develops are accompanied with interactive SketchPad diagrams. He covers basic right angle trig and then does a nice job developing both the Law of Sines and the Law of Cosines. Another plus is a great collection of problems, many of which come from Edwin S. Crawley's *One Thousand Exercises in Plane and Spherical Trigonometry*, written in 1914. Included with the problems are both hints and solutions. Here is a sample.

> **214.** *At two points 65 feet apart on the same side of a tree and in line with it, the angles of elevation of the top of the tree are 21° 19′ and 16° 20′. Find the height of the tree.*
>
> http://www.clarku.edu/~djoyce/trig/right.html

The site also contains translations of Euclid's *Elements*, a short course on the complex numbers, an introduction to compass geometry, some fun dissection puzzles, and a nicely illustrated introduction to the geometry of wallpaper patterns.

Graph Theory Tutorials:
[http://www.utm.edu/departments/math/graph/index.html] If you would like to include a unit on Graph Theory suitable for a high school geometry course, Chris Caldwell at the University of Tennessee-Martin has put together a series of tutorials which are easy to use, thorough, and cover the basics. I have used them with my classes and recommend them highly.

Convergence
[http://mathdl.maa.org/convergence/1/] The Mathematical Association of America's Convergence site is a great source of historical material. It also has a problem section that you can search by subject. I use it to come up with interesting problems for my geometry class that I can use as Problems of The Week. All the problems are historic in nature and each is referenced by date and source. **Figure 2** is an example.

Useful Software and Applets

GeometryGames
[www.geometrygames.org]: This site is maintained by Jeff Weeks, a mathematician and author of the very readable *Shape of Space*. Jeff's interests include both the topology of the universe and improving the teaching of geometry in our schools. The site includes some fascinating games where one can play chess and tic-tac-toe on different types of surfaces. For example, one can play tic-tac-toe on the surface of a torus. There are two other features that should appeal to geometry teachers. The first is a program called Kaleidotile. It is free, can be downloaded from the site, and runs on both PCs and Macs. It allows you to construct many of the Archimedean or semi-regular polyhedra on screen using truncation. The sequence of images showing the progression from an icosahedron, to a truncated icosahedron (a.k.a. a soccer ball or a Buckyball) to an icosadodecahedron shown in **Figure 3** was created using the program.

If you are interested in wallpaper design and showing your students that in the plane there are seventeen different ways one can create a wallpaper pattern that will tessellate, be sure to download and try KALI, which can also be found on this site. It is easy and fun to use. I used it to create the pattern in **Figure 4.**

GeoGebra
[http://www.geogebra.org/cms/] GeoGebra is a free geometric construction applet that is quite similar to both Geometer's SketchPad and Cabri II. It allows you to do all the basic geometric constructions and more. What distinguishes it from the two packages just mentioned is that as you define lines and circles, it gives you their equations. For example, in **Figure 5**, I have constructed a triangle and its incircle. On the left are the equations of the three sides, the three angle bisectors, and the incircle. If you are doing a coordinate geometry unit, this applet could be very useful. I intend to use it a lot in the coming year. If you want to learn more, the Mathematical Association of America's

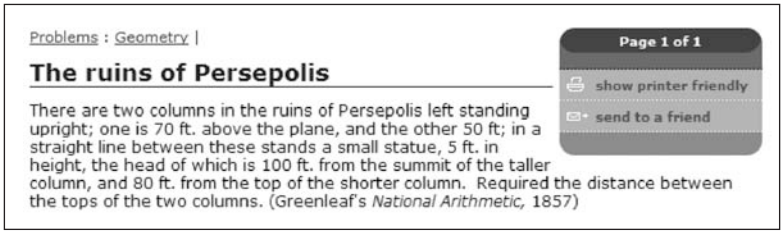

FIGURE 2.
http://mathdl.maa.org/convergence/1/?pa=content&sa=viewDocument&nodeId=998

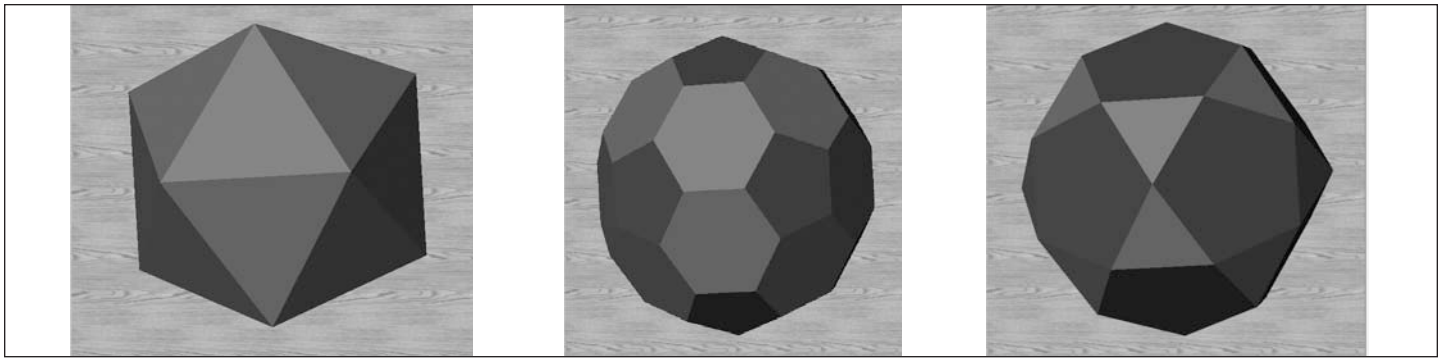

FIGURE 3.

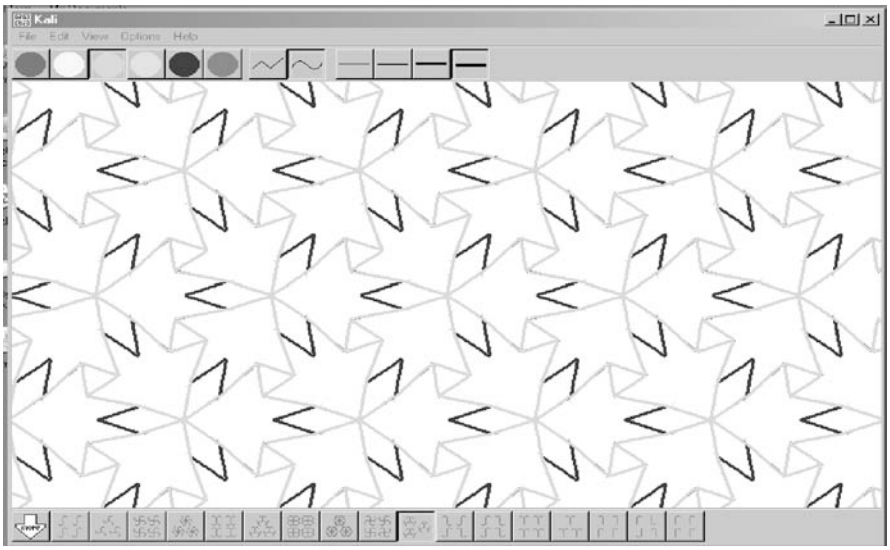

FIGURE 4.

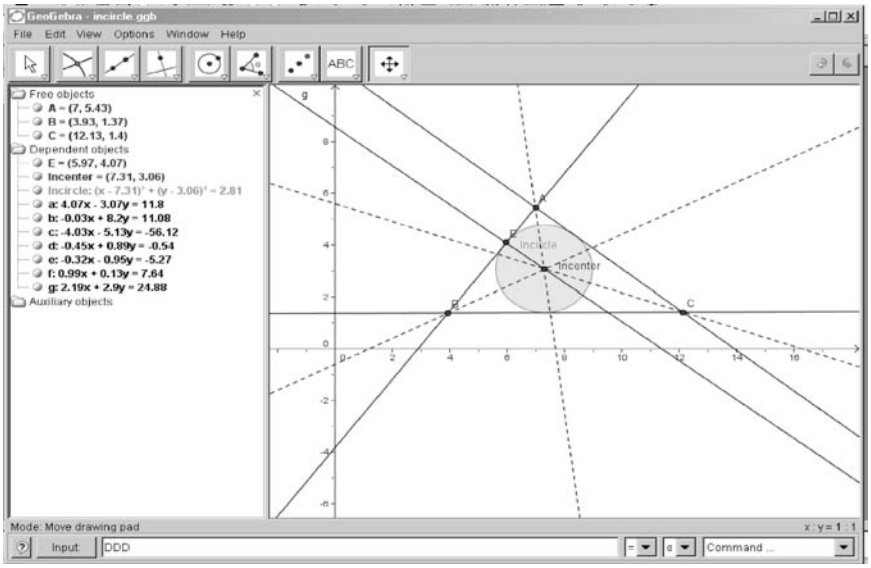

FIGURE 5.

MathDL site has a good GeoGebra article that can be downloaded from http://www.maa.org/joma/Volume7/Hohenwarter/index.html.

Google SketchUp:
[http://www.sketchup.com] This is architectural software that is available for free. It is also an easy to use three-dimensional construction program that has many features that should appeal to anyone who teaches geometry. It is easy to construct a variety of 3-D figures such as prisms/cylinders, pyramids/cones, and lots of different polyhedra. It has all of the basic two- and three-dimensional transformation tools such as translation, rotation, and dilation and has both angle and linear measurement tools. The site also contains a series of tutorials that are well done and easy to use. To download a copy, go to http://www.sketchup.com. Ideas on how to use this wonderful piece of software in the math classroom can be found on my Website at www.zebragraph.com.

There are many more sites that I have found useful but there just isn't space to discuss them all. If you are looking for information about a particular geometric topic or have a site that you have found useful please email me at jchoate@groton.org, and I will both do what I can to help and add whatever you found or I found and add it to the site. ❑